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Ferroelectric Space Groups

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Abstract

The 440 ferroelectric space groups, viz the Heesch-Shubnikov (magnetic) space groups, which are symmetry groups of ferroelectric electric-dipole arrangements in crystals, are derived and tabulated. By considering automorphisms induced by the automorphisms of the discrete space-time group, we show that although ferroelectric, ferromagnetic and ferrocurrent point groups all number 31, the number of ferroelectric space groups differs from 275, which is that of both ferromagnetic and ferrocurrent space groups.

I. Introduction

Which of the 1651 Heesch-Shubnikov (magnetic) space groups are ferroelectric space groups, that is, symmetry groups of ferroelectric electric-dipole arrangements in crystals? Neronova & Belov (1960) have tabulated a list of 275 Heesch-Shubnikov space groups, which they call ferroelectric space groups. These authors, however, do not take into account the fact that the action of elements of Heesch-Shubnikov groups on a polarization P cannot be arbitrarily defined within the usual electromagnetic theory based on Maxwell's equations. Their choice of action is incompatible with that theory and consequently their list of 275 Heesch-Shubnikov space groups is not that of the ferroelectric space groups. With a choice of action that is compatible with that theory, Cracknell (1975) has stated that the number of ferroelectric space groups is 275, Schwarzenberger (1984) gives it as 265. No tabulation is given by either of these

In § II we shall review the action of the discrete space-time group on electromagnetic quantities. We then derive and tabulate the 440 ferroelectric space groups-the Heesch-Shubnikov space groups that are symmetry groups of ferroelectric electric-dipole arrangements in crystals. In § III we discuss why, although the same number, 31, is the number of ferroelectric, ferromagnetic and ferrocurrent point groups, the number of ferroelectric space groups is different from the number, 275, of both ferromagnetic and ferrocurrent space groups.

II. Ferroelectric space groups

Let $\mathcal{U} = 1$, $\overline{1}$, 1', $\overline{1}'$ denote the discrete space-time group consisting of the identity 1, space inversion $\overline{1}$,

Table 1. Character table of the discrete space-time group \mathcal{U}

Basis functions of the irreducible representations, on the right, are given in terms of the charge density ρ and the components of polarization P, magnetization M and current density J.

•	
1 1 1 ρ	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
1 1 -1 -1 M_x , M_y , M_z	
1 -1 -1 1 J_x, J_y, J_z	

Table 2. The thirty-one ferroelectric point groups

1	1'		
2	21'	2'	
m	m1'	m'	
mm2	mm21'	m'm'2	m'm2'
3	31'		
3 m	3 m 1'	3 m'	
4	41'	4'	
4mm	4mm1'	4m'm'	4' m' m
6	61'	6′	
6mm	6mm1'	6m'm'	6'm'm

time inversion 1', and $\bar{1}'$, the product of space inversion and time inversion. Further, let P, M, J and ρ denote the four quantities polarization, magnetization, current density and charge density, respectively, that appear in Maxwell's equations. These quantities can be classified according to the symmetry operations that are the elements of \mathcal{U} (Ascher, 1966). In Table 1 we give the character table of \mathcal{U} and classify the four quantities that appear in Maxwell's equations according to irreducible representations of the group \mathcal{U} . This classification and the symmetry operations of \mathcal{U} on these quantities follows from the assumption that the charge density is invariant under \mathcal{U} and the covariance of Maxwell's equations under \mathcal{U} (Opechowski, 1985).

It follows from Table 1 and the vector properties of P, M and J that the maximal symmetry group of a polarization vector P is $\infty m1'$, of a magnetization vector M, ∞/m m', and of a current density vector J, ∞/m' m. To determine which Heesch-Shubnikov groups are ferroelectric space groups, one determines which Heesh-Shubnikov groups have a point group that is a subgroup of $\infty m1'$. (In a similar manner one finds all ferromagnetic and ferrocurrent space groups by determining which Heesch-Shubnikov groups have a point group that is a subgroup of ∞/m m' or

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Table 3. The four hundred and forty ferroelectric space groups

The table is subdivided into subtables, all groups in one subtable having the same point group listed at the top of the subtable. We list the groups in the notation of Opechowski & Guccione (1965) and, if it is different, on the right give the notation of Belov, Neronova & Smirnova (1957).

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	1		m21'	$A_P b' a' 2 P_B nc 2$	3	4mm	P4'2n'm
P1		$P_{2c}mm2$	P_cmm2	F_C mm2 C_A mm2	P3	P4mm	P4'2nm'
	1'	$P_{2a}mm2$	P_amm2	$F_A mm2 A_C mm2$	$P3_1$	P4bm	P4'c'c
D 1	P_s 1	$P_C mm2$	$C_a mm2$	$F_C mm'2' C_A mc2_1$	P32	P4 ₂ cm	P4'cc'
$P_{2s}1$	rsı	$P_A mm2$	A _c mm2	$F_C m' m' 2 C_A cc 2$	R3	P4 ₂ nm	P4'n'c
P11'			F_smm2	$F_A m' m 2'$ $A_C b m 2$		P4cc	P4'nc'
	2	P_Fmm2			31′	P4nc	P4'2m'c
P_2		$P_{2c}mm'2'$	P_cmc2_1	$F_A mm'2' A_C ma2$	$P_{2c}3$ $P_{c}3$	P4 ₂ mc	
$P2_1$		$P_{2c}m'm'2$	P_ccc2	$F_A m' m' 2 A_C ba 2$	$P_{2c}3_2$ P_c3_2		P4'2mc'
C2		$P_{2a}m'm'2$	P _a ma2	$I_Pmm2 P_Imm2$	$P_{2c}3_1$ P_c3_1	$P4_2bc$	P4'2b'c
CZ		$P_A m' m' 2$	A_cbm2	$I_Pmm'2' P_Imn2_1$	R_R^{201} R_I^{3}	I4mm	$P4_2'bc'$
	21'	$P_{2a}mc2_1$	$P_a mc2_1$	$\hat{I}_P m' m' 2 P_I n n 2$	P31'	I4cm	I4'm'm
$P_{2a}2$	$P_a 2$	$P_{2b}mc2_1$	P_bmc2_1	I _p ba2 P _t cc2		I4 ₁ md	I4'mm'
P_{2b} 2	$P_b 2$	$P_C mc2_1$	$C_a mc2_1$	$I_P ba'2'$ $P_I ca2_1$	P3 ₁ 1'	I4 ₁ cd	I4'c'm
P_C^{2} 2	$\tilde{C_a}$ 2				P3 ₂ 1'	4mm1'	I4'cm'
$P_{2b}2'$	$P_b 2_1$	$P_{2a}mc'2'_1$	P_amn2_1	$I_P b' a' 2 P_I b a 2$	R31'		I4' ₁ m'd
		$P_{2b}m'c'2_1$	$P_a ca 2_1$	$I_P ma2 \qquad P_I ma2$	3 m	$P_{2c}4mm$ $P_{c}4mm$	
$P_{2a}2_1$	$P_a 2_1$	$P_{2a}cc2$	P_acc2	$I_P m'a2' P_I na2_1$	P3m1	P_P4mm P_C4mm	I4' ₁ md'
$C_{2c}2$	C_c2	$P_{C}cc2$	C_acc2	$I_P ma'2' P_I mc2_1$	P31m	P_I 4mm I_c 4mm	$I4_1'c'd$
C_P2	$P_C 2$	$P_{2b}c'c2'$	P_bnc2	$I_P m' a' 2 P_I n c 2$	P3c1	$P_{2c}4'm'm$ $P_{c}4_{2}cm$	$I4'_1cd'$
C_P2'	$P_C 2_1$	$P_{2b}ma2$	$P_b ma2$	Pmm21'	P31c	$P_{2c}4'mm'$ $P_{c}4_{2}mc$	6
P21'		$P_{2c}ma2$	P_cma2	Pmc2 ₁ 1'		$P_{2c}4m'm'$ $P_{c}4cc$	P6
$P2_{1}1'$			$A_c ma2$	Pcc21'	R3m	$P_P4'mm'$ P_C4bm	P6 ₁
C21'		$P_A ma2$			R3c		
	0/	$P_{2b}m'a2'$	P_aba2	Pma21'	3m1'	$P_I4m'm'$ I_c4cm	P6 ₅
	2′	$P_{2c}m'a2'$	$P_c ca2_1$	Pca2 ₁ 1'	$P_{2c}3m1$ $P_{c}3m1$	$P_{2c}4bm$ $P_{c}4bm$	P6 ₂
P2'		$P_{2c}ma'2'$	P_cmn2_1	Pnc21'	$P_{2c}3m'1$ $P_{c}3c1$	$P_{2c}4'b'm$ $P_{c}4_{2}nm$	$P6_4$
$P2'_1$		$P_{2c}m'a'2$	$P_c nc2$	Pmn2 ₁ 1'	$P_{2c}31m$ $P_{c}31m$	$P_{2c}4'bm'$ $P_{c}4_{2}bc$	P6 ₃
C2'		$P_A m' a' 2$	A_cba2	Pba21'	P 21/ P 21	$P_{2c}4b'm'$ $P_{c}4nc$	(1)
	m	$P_{2b}ca2_1$	$P_b ca 2_1$	Pna2 ₁ 1'	$P_{2c}31m'$ $P_{c}31c$	$P_P 4_2 cm$ $P_C 4_2 mc$	61'
D	m	$P_{2b}c'a'2_1$	$P_b na2_1$	Pnn21'	$R_R 3m \qquad R_I 3m$	$P_P 4_2' cm'$ $P_C 4_2 bc$	$P_{2c}6$ $P_{c}6$
Pm					$R_R 3m'$ $R_I 3c$	P_14_2 nm I_c4_1 md	$P_{2c}6'$ $P_{c}6_{3}$
Pc		$P_{2a}nc2$	P _a nc2	Cmm21'	P3m11'		$P_{2c}6_2$ P_c6_1
Cm		$P_{2a}nc'2'$	P _a nn2	$Cmc2_11'$	P31m1'	$P_1 4_2 n' m' I_c 4_1 cd$	$P_{2c}6_2^7$ P_c6_4
Cc		$P_{2b}mn2_1$	P_bmn2_1	Ccc21'	P3c11'	P_P4cc P_C4cc	$P_{2c}6_4$ P_c6_2
	m1'	$P_{2b}m'n2'_1$	P_a na 2_1	Amm21'	P31c1'	$P_P 4'cc'$ $P_C 4nc$	D 6' D 6
D		$P_{2c}ba2$	P_cba2	Abm21'		$P_P 4_2 mc$ $P_C 4_2 cm$	$P_{2c}6'_{4}$ $P_{c}6_{5}$
$P_{2a}m$	$P_a m$	$P_{2c}^{zc}b'a2'$	$P_c na2_1$	Ama21'	R3m1'	$P_P 4_2' mc' P_C 4_2 nm$	P61'
$P_{2b}m$	$P_b m$	$P_{2c}b'a'2$	P _c nn2	Aba21'	R3c1'	I_P4mm P_I4mm	P6 ₁ 1'
$P_C m$	$C_a m$	$P_{F}nn2$	F _s dd2	Fmm21'	3 <i>m</i> ′	$I_P4'm'm$ P_I4_2nm	P6 ₅ 1'
$P_{2c}m'$	$P_c c$				P3m'1	$I_P4'mm'$ P_I4_2mc	P6 ₂ 1'
$P_{2a}c$	$P_a c$	$C_{2c}mm2$	C_cmm2	Fdd 21'	P31m'		P6 ₄ 1'
$P_{2b}c$	P_bc	C_Pmm2	P_cmm2	Imm21'	P3c'1	$I_P4m'm'$ P_I4nc	P6 ₃ 1'
P_{C}^{c}	$\tilde{C_a}c$	C_Imm2	I_cmm2	Iba21'	P31c'	I_P4cm P_I4bm	6'
$C_{2c}m$	$C_c m$	$C_{2c}m'm2'$	C_cmc2_1	Ima21'		$I_P 4'c'm P_1 4_2 cm$	
	$P_C m$	$C_{2c}m'm'2$	$C_c cc2$	m'm2'	R3m'	$I_P 4'cm' P_I 4_2bc$	P6'
$C_P m$		$C_P m' m 2'$	$P_C ma2$	Pm'm2'	R3c'	$I_P4c'm'$ P_I4cc	P6' ₁
$C_{2c}m'$	$C_c c$	$C_P m' m' 2$	$P_C ba2$	$Pm'c2'_1$	4	P4mm1'	$P6_5^{i}$
$C_P m'$	$P_A c$	$C_{I}m'm2'$	I _c ma2	Pmc'2'	P4	P4bm1'	$P6_2^7$
$C_{P}c$	$P_C c$		I _c ba2	Pc'c2'	P4 ₁	P42cm1'	$P6_4^7$
Pm1'		$C_I m' m' 2$			P4 ₂		P6' ₃
Pc1'		C_pmc2_1	P_Cmc2_1	Pm'a2'		P42nm1'	
Cm1'		$C_P m' c 2_1'$	$P_C ca2_1$	Pma'2'	P4 ₃	P4cc1'	6 <i>mm</i>
Cc1'		$C_Pmc'2'_1$	P_Cmn2_1	$Pc'a2'_1$	<i>I</i> 4	P4nc1'	P6mm
CCI	,	$C_P m' c' 2_1$	P_C na 2_1	Pca'2'	$I4_1$	$P4_2mc1'$	P6cc
	m'	$C_{P}cc2$	$P_{C}cc2$	Pn'c2'	41′	$P4_2bc1'$	P6 ₃ cm
Pm'		$C_P c' c 2'$	$P_C nc2$	Pnc'2'	$P_{2c}4$ $P_{c}4$	I4mm1'	P6 ₃ mc
Pc'		$C_{\mathbf{P}}c'c'2$	P_C nn2	$Pm'n2'_1$	P_P^{2c} P_C^{c}	I4cm1'	6 <i>mm</i> 1′
Cm'		$A_{2a}mm2$	A_amm2	Pmn'2'		I4, md1'	
Cc'				Pb'a2'	P_14 I_c4	I4 ₁ cd1'	$P_{2c}6mm$ $P_{c}6mm$
	mm2	A_Pmm2	$P_A mm2$		$P_{2c}4'$ $P_{c}4_{2}$		$P_{2c}6'm'm$ $P_{2c}6_3cm$
D	111114	$A_I mm2$	I _a mm2	Pn'a2' ₁	$P_P4_1 \qquad P_C4_1$	4m'm'	$P_{2c}6'mm'$ $P_{c}6_{3}mc$
Pmm2		A _{2a} mm'2'	A _a ma2	Pna'2' ₁	$P_{2c}4_2$ P_c4_1	P4m'm'	$P_{2c}6m'm'$ $P_{c}6cc$
$Pmc2_1$		Apm'm2'	P_Bmn2_1	Pn'n2'	$P_P4_2 \qquad P_C4_2$	P4b'm'	P6mm1'
Pcc2		$A_Pmm'2'$	P_Amc2_1	Cm'm2'	P_14_2 I_c4_1	P42c'm'	P6cc1'
Pma2		$A_P m' m' 2$	$P_A nc2$	Cm'c2' ₁	$P_{2c}4_2'$ P_c4_3	$P4_2n'm'$	P63cm1'
$Pca2_1$		$A_I m' m' 2$	$I_a ma2$	$Cmc'2'_1$	$P_P 4_3 \qquad P_C 4_3$	P4c'c'	P63mc1'
Pnc2		$A_{2a}bm2$	A_abm2	Cc'c2'	I_P4 P_I4	P4n'c'	-
$Pmn2_1$		A_pbm2	$P_B ma2$	Am'm2'		$P4_2m'c'$	6m'm'
Pba2		A_lbm2	I _b ma2	Amm'2'	I_P4' P_I4_2		P6m'm'
Pna2 ₁		$A_1 o m_2$ $A_{2a} b' m' 2$		Ab'm2'	$I_P4_1 \qquad P_I4_1$	$P4_2b'c'$	P6c'c'
Pnn2					$I_P 4_1'$ $P_I 4_3$	I4m'm'	P6 ₃ c'm'
		$A_Pb'm2'$	P_Bmc2_1	Abm'2'	P41'	I4c'm'	$P6_3m'c'$
Cmm2		$A_Pbm'2'$	$P_B ca2_1$	Am'a2'	P4 ₁ 1'	$I4_1m'd'$	6'm'm
Cmc2 ₁		$A_pb'm'2$	P_Acc2	Ama'2'	P4 ₂ 1'	$I4_1c'd'$	P6'm'm
Ccc2		$A_Ib'm'2$	I_aba2	Ab'a2'	P4 ₃ 1'	4'm'm	
Amm2		$A_P ma2$	$P_A ma2$	Aba'2'	I41'	P4'm'm	P6'mm'
Abm2		$A_Pm'a2'$	P_A^{\cap} na 2_1	Fm'm2'	I4,1'		P6'c'c
Ama2		$A_P ma'2'$	$P_A mn2_1$	Fd'd2'		P4'mm'	P6'cc'
Aba2		$A_pm'a'2$	$P_A nn2$	Im'm2'	4′	P4'b'm	P6'3c'm
Fmm2					P4'	P4'bm'	P6'3cm'
		A_pba2	P_Aba2	Ib'a2'	P4' ₁	P4'2c'm	P6'3m'c
Fdd2		$A_Pb'a2'$	$P_A ca2_1$	Im'a2'	P4'2	$P4_2^7cm'$	P6'mc'
Imm2		A _P ba'2'	$P_B na2_1$	Ima'2'	$P4_3^7$	-	1 03/110
Iba2					I4'		
Ima2					Ĩ4' ₁		
					^ '1		

Table 4. Automorphisms of the discrete space-time group \mathcal{U}

An element in the *i*th row of the left-hand column is mapped under the automorphism A_j in the *j*th column of the top row into the element of \mathcal{U} at the intersection of the *i*th row and *j*th column.

	A_0	A_1	A_2	A_3	A_4	A_5
1	1	1	1	1	1	1
ī	ī	ī	ī'	1'	ī'	1'
1′	1'	ī'	1′	ī	ī	ī'
ī'	ī′	1'	ī	ī'	1'	ī

 ∞/m' m.) The point groups of Heesch-Shubnikov groups that are subgroups of $\infty m1'$ -the ferroelectric point groups-are listed in Table 2.

In Table 3 we list the 440 ferroelectric space groups. These are all the Heesch-Shubnikov groups whose point group is one of the ferroelectric groups (listed in Table 2). Table 3 has been subdivided into subtables, all ferroelectric groups with the same ferroelectric point group appearing in one subtable.

There are 275 ferromagnetic space groups (Neronova Belov, 1960; Opechowski & Guccione, 1965) and 275 ferrocurrent space groups. The latter have been listed by Neronova & Belov (1960) but are called by them ferroelectric space groups. This nomenclature is incompatible with the usual electromagnetic theory based on Maxwell's equations, and is misleading as it does not give the correct number of ferroelectric space groups. Of the 440 ferroelectric space groups listed in Table 3, 68 are non-magnetic space groups F, 68 are of the form F1'and, using the notation of Opechowski & Guccione (1965), 129 are magnetic groups M_T and 175 are magnetic groups M_R . We note that the sum of the first three types is 265, the number of ferroelectric space groups given by Schwarzenberger (1984).

III. Numerology

There are 31 ferroelectric point groups, 31 ferromagnetic point groups and 31 ferrocurrent point groups (Ascher, 1966; Cracknell, 1972; Kopsky, 1976; Ascher & Janner, undated). There are 275 ferromagnetic space groups, 275 ferrocurrent space groups and 440 ferroelectric space groups. To understand why the number of ferroelectric, ferromagnetic and ferrocurrent point groups is the same, and why the number of ferromagnetic and ferrocurrent space groups is the same, though different from that of ferroelectric space groups, requires considering the automorphisms of the discrete space-time group \mathcal{U} .

The six automorphisms A_1 , i = 0, 1, 2, 3, 4, 5, of the discrete space-time group \mathcal{U} are listed in Table 4 (Kopsky, 1976). These automorphisms of \mathcal{U} induce automorphisms of the group $R_+(3) \times \mathcal{U}$, where $R_+(3)$ is the group of all proper three-dimensional rotations: Let $A_i[u]$ denote the element of \mathcal{U} into which the

Table 5. The set of all ferroelectric (FE), ferromagnetic (FM) or ferrocurrent (FC) point groups given in the ith row of the left-hand column is mapped under the automorphism A_j in the jth column of the top row into a set of FE, FM or FC point groups according to the entry at the intersection of the ith row and jth column

	A_0	A_1	A_2	A_3	A_4	A_5
FE	FE	FC	FE	FM	FM	FC
FM	FM	FM	FC	FE	FC	FE
FC	FC	FE	FM	FC	FE	FM

element u of \mathcal{U} is mapped under the automorphism A_i . The mapping $A_i[R_+u] = R_+A_i[u]$ then defines an automorphism of $R_+(3) \times \mathcal{U}$. Under these automorphisms of $R_{+}(3) \times \mathcal{U}$, the set of all ferroelectric point groups is mapped into sets of point groups, which, depending on the automorphism A_{in} are sets of distinct ferroelectric, ferromagnetic or ferrocurrent point groups. The same is true for the set of all ferromagnetic point groups and the set of all ferrocurrent point groups. In Table 5 we show how each set of all ferroelectric, ferromagnetic and ferrocurrent point groups is mapped under each of the automorphisms of $R_+(3) \times \mathcal{U}$ induced by the automorphisms A_i of U. It follows that the number of ferroelectric, ferromagnetic and ferrocurrent point groups is the same. For example, the automorphism A_2 maps the set of all ferromagnetic point groups into a set of distinct ferrocurrent point groups and simultaneously the set of all ferrocurrent point groups into a set of distinct ferromagnetic point groups. Consequently, the number of ferromagnetic and ferrocurrent point groups is the same.

However, only the identity automorphism A_0 and the automorphism A_2 of $\mathcal U$ induce automorphisms of the group $\mathscr E_+(3) \times \mathcal U$, where $\mathscr E_+(3)$ is the proper three-dimensional Euclidian group. From the automorphism induced by the automorphism A_2 of $\mathcal U$, it follows that the number of ferromagnetic and ferrocurrent space groups is the same. As there are no other such induced automorphisms, we conclude that the number of ferroelectric space groups may be (and, as we have shown, is) different.

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Neutron Diffraction Investigation of the Nuclear and Magnetic Extinction in MnP

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Abstract

The absolute values of the reflecting powers ρ are measured for the 200 and $2 \pm \tau$, 0, 0 set of magnetic and nuclear reflections in the helimagnetic phase of a good-quality crystal of MnP as a function of its thickness. Severe and very different extinction effects are observed for the magnetic and nuclear reflections $(y_{\text{magnetic}} \sim 0.4, y_{\text{nuclear}} \sim 0.02$ for the largest thickness). This corresponds to the spectacular result that the magnetic reflecting powers ρ_{\pm} are twice as big as the nuclear one ρ_N , in spite of the fact that the scattering cross sections $|F_{\pm}|^2$ are about ten times smaller than the nuclear $|F_N|^2$. The nuclear results appear consistent with dynamical theory while the magnetic ones are not. They can be explained by Zachariasen's type II secondary extinction model based on the chirality domain pattern. The same measurements were performed in the ferromagnetic phase, yielding $y_{\text{ferro}} \approx 0.03$. A model using the relative sizes of the ferromagnetic and chirality domains is presented.

I. Introduction

The basic publication on extinction for the neutron case, within the framework of the mosaic model, is now nearly forty years old (Bacon & Lowde, 1948). Since then most of the improvements introduced to correct the extinction of the intensities diffracted by a single-crystal sample originate from the theory based on the Darwin energy transfer equations

worked out by Zachariasen (1967). This theory was modified to take into account the anisotropy of the extinction by Coppens & Hamilton (1970) and Thornley & Nelmes (1974). The formalism was reconsidered and improved by Cooper & Rouse (1970) and Becker & Coppens (1974a, b) in order to apply it to spherical or ellipsoidal crystals, the theory being extended to non-spherical crystals with anisotropic extinction by Becker & Coppens (1975).

The main limitation of Zachariasen's theory is in its kinematical approach to the scattering, as pointed out by Werner (1969, 1974): the coherence of the transmitted and diffracted beams is not taken into account, and so this method does not appear to be suitable for correction for severe primary extinction. Another approach, starting from the dynamical theory of diffraction, was worked out for distorted crystals by several authors (Klar & Rustichelli, 1973; Gronkowski & Malgrange, 1984; Kulda, 1984), but mainly by Kato (1976), who has partially reconciled the two approaches. He shows that for optical coherence lengths smaller than the extinction distance Λ the new treatment leads to the usual coupling equations. Kato (1980) has also developed a consistent statistical theory of extinction covering the whole range of crystal quality from perfect (dynamical theory) to ideally imperfect (kinematical approximation). The results of this last theory have recently been compared to those of previous ones (Becker & Dunstetter, 1984) and experimentally tested using polarized neutrons (Guigay, Schlenker, Baruchel & Schweizer, unpublished).

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